

Polarities – Projective Geometry

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Chapter 5 in: Guus van der Bie and Machteld Huber (eds), *Foundations of Anthroposophical Medicine*, Floris Books, 2003, ISBN 0-86315-417-4.

In the previous chapter, the figure of the lemniscate was used several times in order to help the reader picture and experience how polarities can be turned inside out. Drawing exercises were also given to clarify the process of thinking one's way through from one polarity into the other. Chapter 4 discussed the polarity between what anthroposophy calls the spiritual world and the material world. Anthroposophy sees these areas as complementing one another, yet forming a unity. In this chapter an attempt will be made to give the reader an experience of the reality of this polarity – and mutual interdependence - by means of geometrical descriptions and representations. This may seem a rather theoretical pursuit, far-fetched and removed from daily reality. Yet these mathematical imaginations are a help to gain access to this realm which is hard to penetrate. Therefore this should be regarded as practice material.

This chapter's approach aims to make clear that the anthroposophical concept of a 'spiritual world' is neither grounded in free associations of the imagination, nor in unclear raptures about 'energy clouds' and the like. We have the capacity to get to know a spatial world of a different kind from the material world with its characteristic measurable quantities. This capacity rests on mathematical thinking, which all of us can use to a certain degree. We can get an initial glimpse of the supersensory world when we try to do this with the kind of clarity which is only granted by mathematical thinking. Mathematics is the only exact science which is totally perspicuous, by virtue of the fact that it is completely unconnected to the senses (it is abstract) and basically cannot error.* Through the inner power of the activity of thinking we can thus transcend the inherent limits of the sense world.

In anthroposophy the level of life, which lies directly next to sense perceptible reality, is called the 'etheric' world.' Although it is invisible, it can be grasped as an idea. The examples dealt with in this chapter aim to get to the beginning level of an initial experience of this etheric world, as it relates to physical-material reality.

* Of course we can make errors, but by the same token we can trace them and acknowledge them.

5.1 Point–circle: turning inside-out

In Chapter 1 of the book *Extending Practical Medicine*, Rudolf Steiner and Ita Wegman described how the *forces of attraction* are characteristic of the physical world. The centre of the earth is the primary place from which this power of attraction works, namely the force of gravity. When we look more closely at several physical phenomena, however, we see that many centers of forces can be at work within the inorganic world. These do not always have to be forces of attraction, but can also be forces of repulsion. The essential point is that in principle one can always point out the *place* where these forces are at work, in other words: they are located in *physical, earthly space*. Even when we're dealing with forces which emanate from heavenly bodies (forces whose source is far away), they are still located within this same space. Even forces which by nature are not really mechanical - such as light, chemical effects, phenomena of warmth, etc. - have a point of origin which can always be located somewhere in space. All these forces of the physical world work on things from the outside.

In the plant world (the organic world) other forces are at work too. Of these Steiner says, however, that they do not emanate from a central point, but radiate from out from the whole of the periphery of the cosmos into this centre. Thus the direction of these forces is diametrically opposed to the forces mentioned earlier. Steiner calls these *etheric forces*, which belong to the ether world, a supersensory world, and the origin of these forces cannot be pinpointed; we can only do that with forces belonging to the physical world. It is these etheric forces which, for example, make it possible that plants grow up, against gravity (contrary to the laws of the physical world). Living nature does not live in conflict with the laws of dead nature, but works in spite of these laws. Generally speaking, we can say that we can distinguish two kinds of mechanisms which are at work, namely those characterized by forces of pressure (in physical space), and others which are characterized by forces of suction (in etheric space). How are these two related? Do they coexist, or are they perhaps far removed from each other?

Let us now look at the significance of such questions for our picture of space; we could very easily pass these questions by, overlooking their implications and not truly experiencing them inwardly. Everything on earth has its place, or is moving from one place to another. In the process of checking how such a mental picture comes into being under the influence of thinking, one will notice that in forming mental pictures there is *no escape from physical space*. I am placed in space together with every object or thing I consider, however far removed, and that includes the living plant. Our mental picturing is therefore apparently bound to physical space.

Now we do have the possibility - which exists within our thinking only - to call on the concept of 'infinity' to take us a step further. We thus indicate, with a word for the time being, that there is a boundary to physical space. Even though this concept may go beyond our powers of imagination, it is a concept nevertheless. Our capacity to form mental pictures is earthbound after all, and cannot take in anything which is not *also* earthly in character.

So what do we do with this statement that there are forces which stream in from out of a cosmic periphery? Where do *they* have their source? After all, we only know diverging forces. We cannot determine the origin of *converging* forces; at most we can point to the place where they take effect. This is disorienting.

We started out with the question, Where does the physical (material) world border onto the spiritual (etheric) world? Are they in fact worlds which lie next to one another? However familiar we may be with the look of the plant which stands in the pot on the windowsill, it lives only partly in the space we are familiar with, namely insofar as it is a physical body. What distinguishes it from the objects in dead nature lies in a supersensory (spiritual) world, of which you cannot just say that it is 'somewhere else.' Entering that world demands the development of a different kind of thinking (Steiner calls it 'pure thinking, *Reines Denken*'), which does not suffer from the

limitations of our earth-bound thinking, tied to the brain. It is possible to do thought exercises which, starting from normal thinking, bring us closer to experiencing the boundary between the physical and the etheric world. Mathematical thinking is eminently suitable to escape physical space. After all, our thoughts themselves, which we use to practice mathematics, cannot be said to exist 'in space' anymore.

In Assignment 1 we see what happens with the circle when the centre moves away from us: it becomes ever bigger and bigger. At last it will be so big, that the distinction between it and a straight line no longer exists. The centre is then at infinity.

We can now continue this process in thought, and why could we not? If the centre continues to move further away, what happens to the circle?

Assignment 1: An exercise in thinking (see Figure 5.)

Imagine a circle, which has a radius and a centre. In thought, position yourself on the circumference and let the centre move away from you. It moves further and further away. What happens to the curvature of the circle?

Of course the centre does have a specific location, but direct your attention especially to the direction in which it moves away from you. The curvature of the circle becomes less and less and we can see the eventual outcome: a straight line! This will only happen when the centre arrives in infinity. From every point on the circumference a radius goes to the centre, so from the place where you are standing you can at least indicate the direction in which that centre must be located. In this case it is perpendicular to the straight line which the circle has now become.

Now consider the following: I in (or 10 m, or 10 km, etc.) further on someone else stands on the same circle. She does the same, she also looks from her point of view at the same centre. Are we now looking at the same point in the direction of infinity? Both persons are oriented in the same direction, their gaze runs parallel!

Repeat the same exercise a few times in your imagination. The point is to experience it! How would it be if somebody were to stand on the circle, but who is facing the centre perpendicular to your own view? Would both orientations still be perpendicular to one another by the time the centre arrives at infinity?

And, as a last example: how would it be if two people would face one another?

Here we need the help of logic. We have seen the movement which the curved line of the circle made. This movement in turn has to continue. What used to be concave will now become convex, and the other way around. This means that the direction into which the centre has vanished will now be opposite. It will approach us from behind and a new circle will come into being, which, as the centre moves further, becomes smaller again.

It is important to note here that the first and the second circle are the same. Their identity is clear: there is a continuous transition from the one to the other. But the similarity stops here, because what used to be inner world in the first circle, becomes outer world in the second one.

The limitless (light) field outside the first circle in turn becomes the limited inner world in the second one.

So far this thinking exercise has mainly led to a reaffirmation and a re-experiencing of what was said about polarities and turning inside out in Sections 4.2 and 4.3. (Compare Assignments 4 and 5 in Section 4.2.5). The point here, however, is to take the mental picture further, in order to come to an experience of the 'border territory' between the physical and the etheric worlds.

As stated before, all forces which emanate from one point in physical space take a diverging course. Taking into account what Rudolf Steiner said about the forces in the plant kingdom, or more generally speaking, the etheric realm, those must be convergent in their direction; they were described above as radiating in from all around the cosmos). The question then becomes where those forces

originate. Their origin could not logically be located in a place which we can imagine concretely; we would not be able to point it out. It would be located in the second circle indicated in the example given for practice. This would be the strict mathematical consequence of the behavior of the diverging lines of force (the radii in the first, upper circle), which became evermore parallel as the centre approached infinity. When the lines of force run exactly parallel, the centre (here re-presenting the origin of forces) lies exactly in infinity. We know that this must be true, even though we cannot imagine it! Now when the centre moves even further than infinity (we are breaking through the boundary of physical space, in other words), the lines of force will of necessity begin to converge if we think this through to the end. In Figure 5.1 we saw how a second circle forms as a consequence, and that the origin of the converging lines of force comes to lie in the centre of this second circle therefore. So the circle can only come into existence when we break through the barrier of the infinite surface (line). And we cannot even imagine infinity, let alone a position which is even further removed! Within our drawing, however, it remains possible to survey all this. The location of the centre of the second can still be indicated, but we must not forget that it would be located under the horizontal line. In the drawing, this horizontal line in fact symbolizes the boundary of the sense perceptible world.

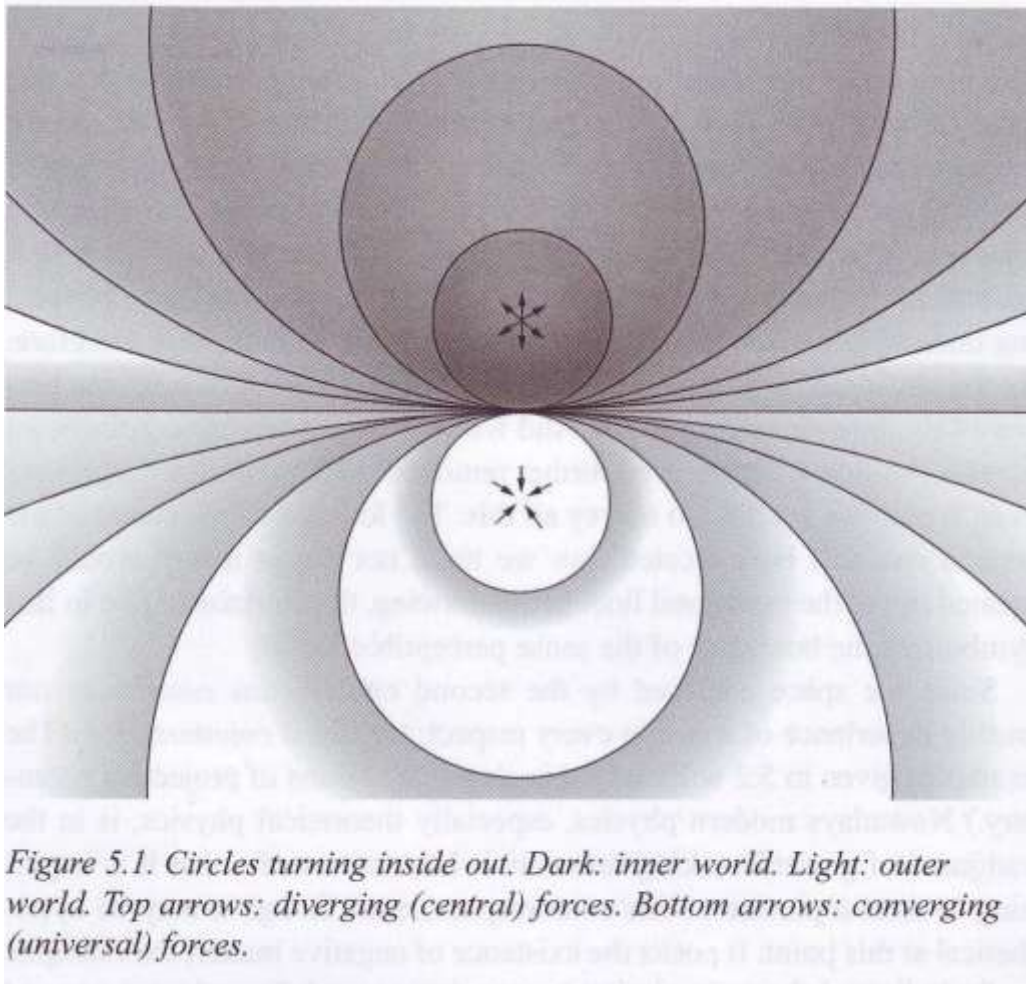


Figure 5.1. Circles turning inside out. Dark: inner world. Light: outer world. Top arrows: diverging (central) forces. Bottom arrows: converging (universal) forces.

Since the space enclosed by the second circle* runs counter to our earthly experience of space in every respect, we call it counterspace. (The examples given in 5.2 will make this clearer by means of projective geometry.) Nowadays modern physics, especially theoretical physics, is in the vanguard of groundbreaking

research. It has come so far that it is beginning to form a picture of this counterspace, even though it may be hypothetical at this point. It posits the existence of negative matter, for example.

As indicated above, in physical space, forces work from the outside and influence one another that way; the sources of strength which are situated in counterspace, however, work from the inside. Therefore they have an internal point of application, and bring substances in line with the laws of the etheric world. This can be observed best in growth processes which are totally subject to physical laws, in contrast to those in living nature. When we follow the growth of a crystal we see that this happens by means of apposition of material, so it accrues on the outside. A living organism grows because of internal growth processes.

We find the origin of etheric forces which ray in: they originate as it were in the centre of the second circle. The horizontal line (see Figure 5.1) separates two spaces from one another, the one lying inside the circle, the other outside. The circles themselves delineate a boundary between two spaces, which cannot be crossed; crossing it is only possible in infinity that is to say in a realm which we cannot picture concretely. We can only determine this boundary mathematically, we have no physical access. This has consequences for the way we have to view the relationship of the etheric body to the physical body in living creatures. The spaces which these bodies occupy do not have topographical boundaries! These boundaries can only be grasped as ideas.

When we inject a substance (for example medicine) subcutaneously into the living body of a human being, we are inclined to view this as introducing that substance into the body. However, this is only correct in one sense. The space outside the skin - physical space, that is - borders directly on the space which is enclosed by the skin, so they form a continuity. By contrast, when we mean etheric space, we are dealing with a discontinuity between the two spaces. The substance only enters inasmuch as we regard the body as belonging to physical space. With a physical injection needle we principally have no direct access to the living part of the body. Injected substance has to be taken into life that is to say into those processes which lie within the domain and guidance of the ether body. Physiology and biochemistry describe how a substance fares when it is introduced into the body. Advanced technology allows us to have detailed knowledge of how that substance is transformed or broken down, where it goes in the body, etc. Please note, however, that the substance under consideration has then been reabsorbed into the physical-material body of the human being.

In current biology and medicine it is said that sodium chloride is subject to the same laws, no matter if it is inside or outside the body. When one researches the processes inside the body only using the knowledge of the laws of physical space, it will also be impossible to escape from this space. However, once a substance has been absorbed by a living organism that means that it is also in etheric space, in counterspace. When a substance has been brought into the body through the skin, i.e. 'here' in the physical worlds, another boundary is simultaneously crossed, namely the one into infinity. In order to assess how a substance influences the body — and that also goes for nutritional substances — one constantly has to be aware of the aspect under consideration. One has to bear in mind whether one has a physical focus, and therefore dealing with central forces, or an etheric (living) focus, dealing with the peripheral forces. In fact, medicine and biology are constantly researching effects originating in counterspace. But these effects are not recognized as such.

*Of course we can imagine the circle in three dimensions, where it would be a sphere.

Therefore they are explained on the basis of knowledge of the material world. This usually succeeds, even though hypotheses are sometimes needed which cannot be verified. Therefore a materialistic worldview never really comes into conflict with manifestations of life. The fact that plants grow upwards can also be reconciled with the laws of thermodynamics. The growth gestures which plants exhibit indicate that they belong to another realm, but such a statement no longer fits in such an interpretative model simply because a concept like 'gesture' does not belong in the realm of matter

5.2 Projective geometry

In the previous paragraph we referred to the world of lifeless and of living nature. In actively completing certain trains of thought which take infinity into account, one can get a first inkling that these two worlds only meet in infinity. Our starting point in carrying out these thought exercises was the familiar space around us, which led us to the initial idea of counterspace, also called negative space. These mental pictures are derived from projective geometry. This is a mathematical method which is relatively young in comparison with other branches of mathematics, one which only began to blossom in the nineteenth century. It lends itself very well to the tracing of formative processes. As a result, one can follow these processes more easily and has the possibility to live into them. Projective geometry is a particularly apt method for elucidating and describing metamorphoses. This geometry deals with phenomena in 'projective space,' i.e. the space in which the special nature of the location of points, lines or planes in infinity is not taken into account as such. So they receive the same treatment as finite points, lines or planes.

Projective geometry can lead to the insight that everything which appears to us as living forms in sense-perceptible space has a necessary parallel in a different world. These living forms can be experienced as 'not of this world,' which necessitates seeking the origins of such forms in a different world. That is to say, a world which lies beyond our earthbound capacities of mental picturing. If this is indeed the case, namely that appearances which I can look at have their correspondences in another realm, it would be worth the trouble to research the relationships between these two 'spaces,' I would then be perceiving this 'something' in another realm from the inside, in contrast to looking at it from the outside, the relationship being that the one realm behaves like the other one turned inside out. For we concluded that the two realms mentioned were qualitatively the opposite of one another. If we imagine the circles in Assignment 1 as spheres in three-dimensional space, we can experience how the first surrounding sphere will ultimately metamorphose into another one, whereby what used to be 'Inside' now becomes 'outside.' This viewpoint will give us an important perspective which allows us to penetrate to the essence of living organisms.

Assignment 2 An exercise in thinking and imagining

Taking Assignment 1 as our starting point, we can take a different initial position by placing ourselves in the middle of the sphere. Now we allow the surrounding sphere to become ever bigger until it approaches infinity. As a result, the curved inside of the sphere which we are looking at will become a plane. This will happen not only in front of us, but also to our left and right. It will also happen behind, above and below us. The result, however, remains a plane!

Repeat this exercise many times and try to experience it as intensively as possible.

With this exercise we have made a groundbreaking discovery: wherever we find ourselves, we can feel surrounded at all times by a plane. We can call this 'the all-embracing cosmic plane.'

Present-day explanations of the genesis of an organism are based on mechanistic laws. In that domain, however, only point-based forces play a role, as we have just seen. These are all surrounded by the cosmic plane at infinity that we have just discovered, so it is 'where the finite ends.' We can surmise that the sources of everything which has living formative capacity lie behind that. There would be the origin of the peripheral universal forces, which we saw were converging, in contrast to the diverging forces of the physical world. Once we know how to escape from the constraints imposed by orthodox models of explanation the way is free to approach the morphology and dynamics (physiological processes) of living beings in a different way.

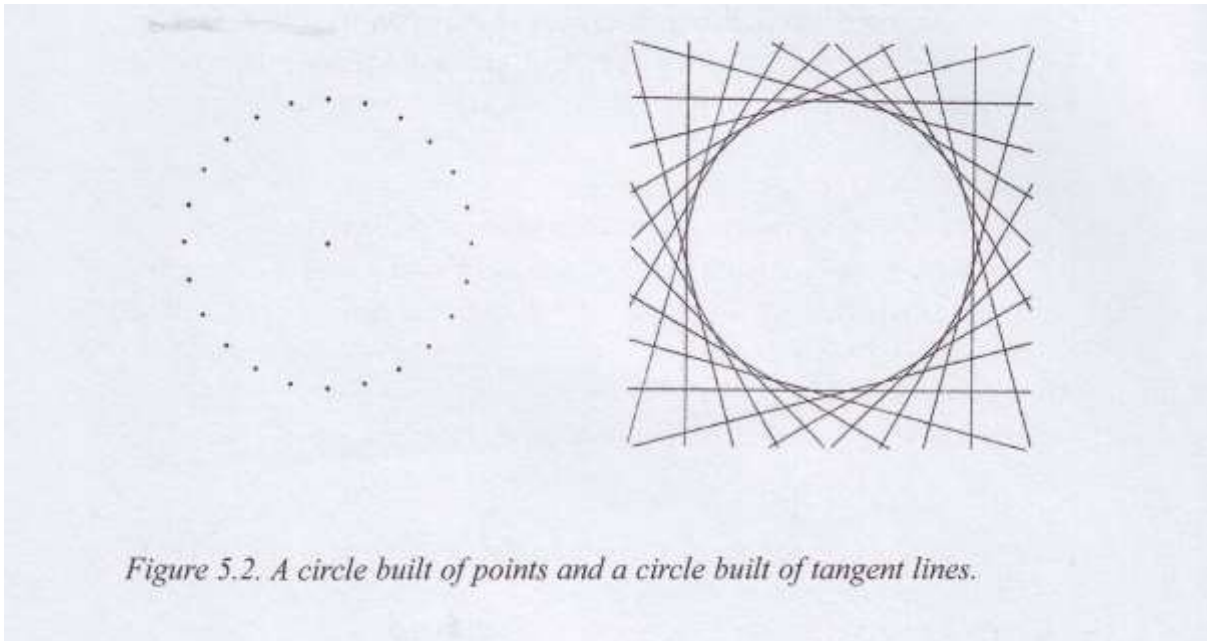
One example is the current embryological tenet that limbs grow out of the body. The final form of the hands would thus come into being because growth processes simply cease. But why would a hand not come into being because the shape of the hand was there first, so growth processes would cease when that shape would determine it? In this case the aforementioned peripheral forces would be exerting their pull. The difference with the current assumption would then be that this shape of the hand is not the product of physical and chemical conditions of cells etc. (which also play in, of course), but a manifestation of a non-material, dynamic formative process. To explain shapes and processes, there are always causes to be indicated in our familiar physical space. However, when unprejudiced observation allows us to see that an arm or hand grows into a final form, could we not postulate that cause and effect may be conceived as reversed in time? So in fact there are two equally valid ways of explaining phenomena, which can be seen as contrasting or parallel explanations:

1. There are intrinsic, programmed, (genetic and epigenetic) growth processes (cause). There is as yet no definitive form, but there is a progression through developmental stages resulting in the shape of the hand (effect).
2. The shape of the hand is there first (cause) and it sets processes in motion (effect) which are brought to an end by the formative principle.

Here we see two diametrically opposed explanations for the same manifestation. They seem to be mutually exclusive, but there is as yet no reason to be tempted to mark either as the only correct one. There is a substantial difference between the two, however. That they can coexist nonetheless is due to the fact that they have been formulated from two different points of view. The first explanation stems from the world of the point-based forces (mechanistic forces), the second from the world of the peripheral forces that stream in. The latter can be taken as a dynamic-creative gesture, a meaningful language which never comes to a standstill. Both principles are in harmony with one another and take each other into account within living organisms. What we meet here, in fact, is a problem which has been known since the Middle Ages: what comes first, the ideal form or the actual matter? It is possible to escape from this dilemma when we take the following into consideration. The physical-material world (the dark area of Figure 5.1) is gripped or penetrated by formative forces originating in the area of the 'all-embracing plane.'

We can learn to see through this with the help of projective geometry. We will now go on to give a few vignettes which do not pretend to be an exact logical argument, but appeal to a common sense of self-evidence which we all share. In this process it will be inevitable to make leaps in thought, and we cannot go into mathematical proof here. For readers who want to penetrate this material further, see the bibliography.

This geometrical method allows us, for example, to construct two kinds of circles. One circle can come into being by drawing an infinite number of points equidistant from a centre. This circle is determined from within, and extends no further than the length of the chosen radius. But likewise a circle can be formed by an infinitude of tangents, coming from infinity and returning there, just touching the circle from the outside. This circle is determined from without. In both cases we see circles as a result, but each one has a totally different origin* (see Figure 5.2).



Projective geometry allows us to penetrate countless phenomena in living nature in a way which differs from conventional methods. This is not Drily true for the observation of definitive forms (morphology), but also for the genesis and change of forms (morphogenesis). In this way it is possible to reach a more essential feeling for and experience of the true nature :)f an organism. An organism lives both in space and in time, or, put in different terms: in actuality. Modern streams in biology, as proposed for example by Sheldrake, Lovelock, and in structuralism, present all-embracing views which conceive of the earth as one organism. Discoveries such as these, point to far-reaching connections.

A typical trait of projective geometry is that this is a discipline dealing with the qualitative aspect of things. It does not express distances in centimeters, nor does it count angles in degrees. All constructions can be carried out with a straightedge and compasses without measurements, in contrast to the more quantitative character of 'ordinary' Euclidean geometry (see the circle built up of points). Projective geometry deals with points, lines and planes which can be seen as projections of other points, lines or planes; hence the name. Dual and polar relationships and definitions form an important basis; an example of this in two-dimensional space is the polarity between a line and a point.

If we imagine this in three-dimensions we would be dealing with a sphere instead of a circle. In that case the tangents will no longer be *lines*, but *planes* coming out of and returning to infinity.

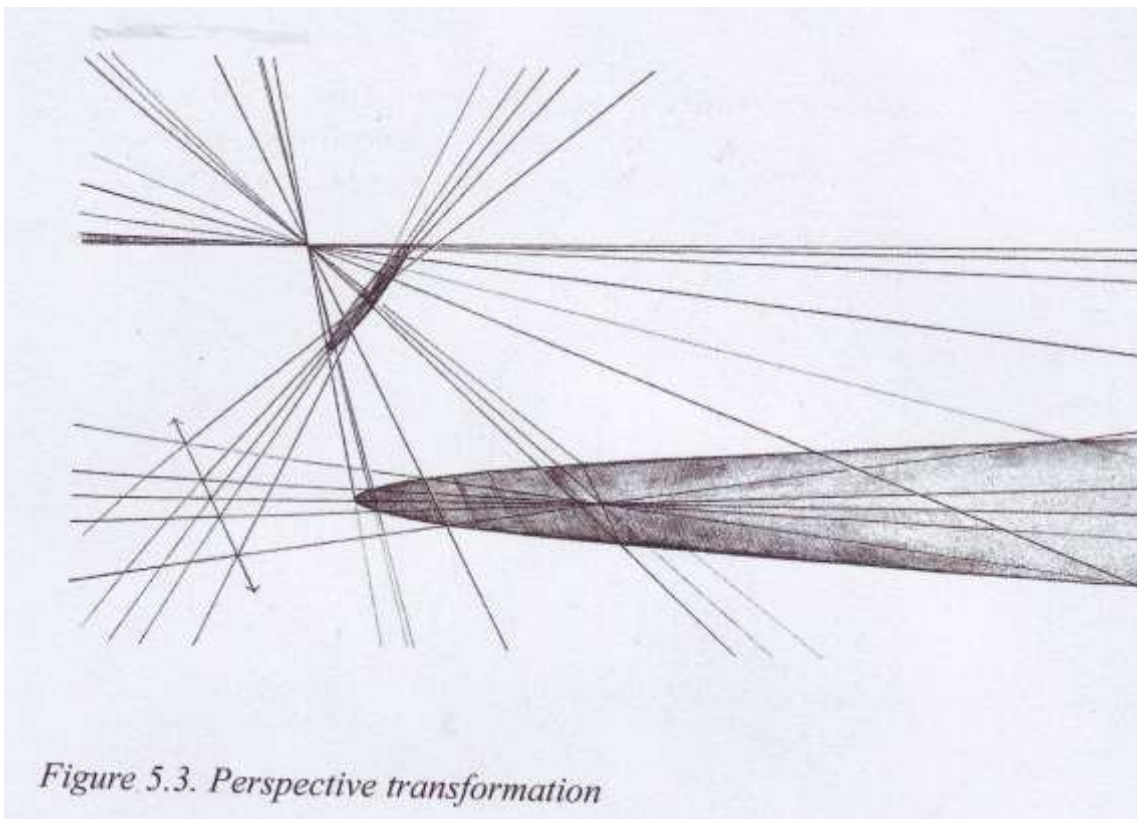
Two lines determine a point (the crossing point) and two points determine a (straight) line.

Or, in three-dimensional space, between line and plane: two intersecting lines determine a plane and two intersecting planes determine a line.

Point and plane are polar opposites, they are mutually exchangeable, whereby the line functions as an intermediary. These are examples of the so-called duality principle (polarity-principle would be a better term for this, but the concept of polarity in projective geometry is used in a more specialized sense). This duality-principle clearly shows that two definitions can both be valid, the only difference being one of perspective. Different standpoints give rise to different definitions.

Figure 5.3 demonstrates the projective character of this type of geometry.

Here we see how a point (formed by a light source for example) throws a shadow of a circle on to a plane positioned at a certain angle to the plane of the circle. The situation in the illustration has been chosen in such a way that the line connecting the point with the upper edge of the circle runs exactly parallel to the projection plane. Under such circumstances a shadow results which has the shape of a parabola. Other positions of the circle in relation to the projection plane will result in ellipses, different circles, hyperbolas, or parabolas. That is to say, they are all conic sections. We can conclude from this that these figures all belong to one family, that they are related and also that they can stem from a single principle.



Investigate what positions give rise to the conic sections mentioned above.

By choosing different initial positions, while keeping the constructions the same, many different curve shapes can be obtained. The relationship between the separate curves is the result of their common origin, as shown for example in Figure 5.4.

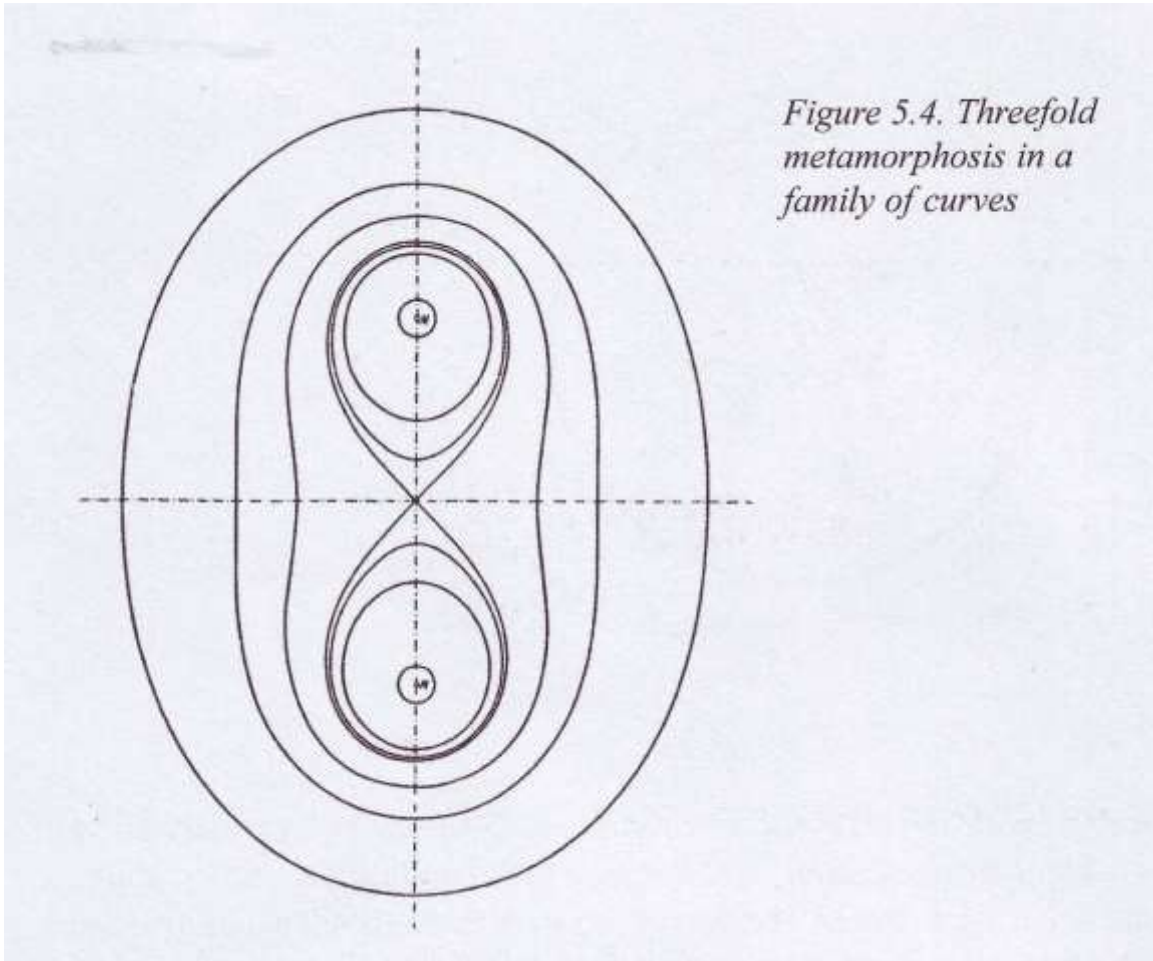


Figure 5.4. Threefold metamorphosis in a family of curves

In Figure 5.4 the so-called curves of Cassini are represented.* They are only six examples out of an infinite possibility of curves, all answering to certain geometric formulas. By choosing different initial positions forms can come into being which are nearly elliptical, and others which show a narrowing on the side. The well-known lemniscate will be recognized as well, it being another member of this family. Furthermore, there are egg-shaped pairs of curves, which form a single curve together in spite of their discontinuity (they are connected through counterspace). One can let this family of curves expand in thought; it will then approach the 'all-embracing plane.' If one lets it shrink, it will disappear into one point (the all-relating point).

Such a family of curves together forms one species, which can express itself in various manifestations, analogous to the classification 'dicotyledon' or 'mammal,' for example. The figures shown here are all still two-dimensional, they are projected onto a plane as it were, but in their perfect forms they are in fact three-dimensional shapes. The pliability of these spatial mathematical figures can be compared to metamorphoses in living nature. Although they have very different manifestations there, they still have things in common; in the one case they correspond to the requirements of geometrical rules, in the other case the requirements of the morphogenetic principle which Goethe calls *Typus* (type). For everybody in physical space a corresponding body can be found in counterspace. In projective geometry this can be shown with constructions, even though they could become very complicated.

* Constructing these curves is quite complicated and demands exact drawing. Interested readers are referred to Louis Locher-Ernst, *Urphänomene der Geometrie* (Basic Phenomena of Geometry), from which this illustration was taken. This book also contains the procedure to construct the curves.

In that case we are dealing with pairs of spatial bodies, which not only have a polar relationship, but are also located in different spaces. With this we introduce the real concept of polarity in projective geometry. This phenomenon of polarity is facilitated by an intermediary, in relation to which the projection is carried out. An example of this is the spherical form,* whereby one spatial body is located in the sphere, the other one outside the sphere (think of Assignment 2 above). Figure 5.5 shows a simple example.

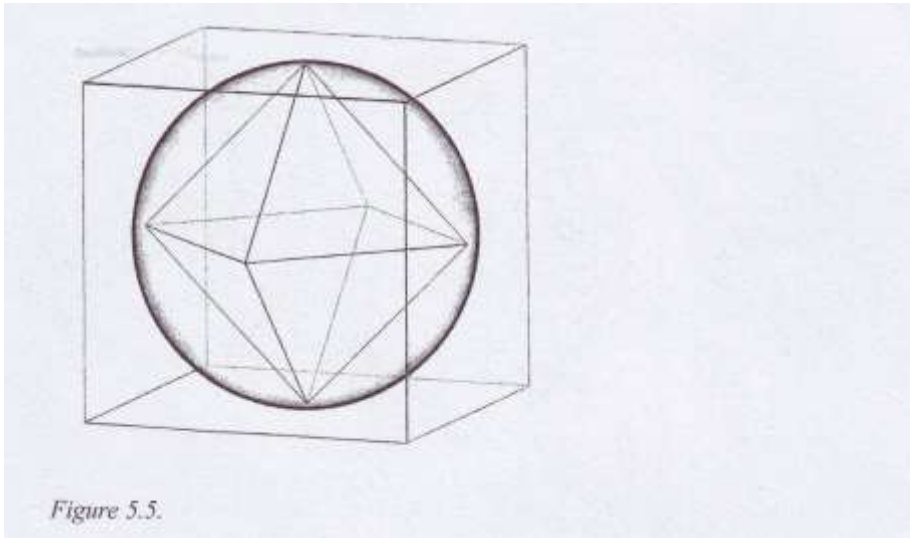


Figure 5.5 shows a cube with inscribed sphere in such a way that this sphere touches the inside of the planes of the cube. Inside the sphere an octahedron can be inscribed. The mutual relationship between the two bodies is such, that the octahedron forms a point where the sphere touches a plane of the cube. Thus the polarity is between plane and point, in accordance with the described duality-principle in projective geometry. The relationship between the two bodies is subject to one law. That implies, that the opposite must hold as well: when one draws the octahedron outside the sphere, there will be a corresponding cube inside the sphere. This goes not only for the bodies which touch the sphere as a plane or a point, but also for larger or smaller bodies** which do not touch the sphere, for which it is an intermediary, however. From Figure 5.6 we can deduce that this relationship also shows a quantitative reciprocity: when the cube decreases, the octahedron increases correspondingly and vice versa. If we take the polarity between point and plane as our starting point, the polar relationship between cube and octahedron retains its validity if we let one of them decrease in size. The other one has to increase correspondingly, as indicated above. Thus basically all kinds of shapes can be translated into their polar counterpart (counter-image). This means that very small organisms, such as the flea, a seed, or the zygote, are likely to correspond to very large counterparts in the dimension of peripheral forces. Through projective geometry, we thus find the same principles which we met before in Sections 3.2 and 3.3, where we found them in a totally different way namely by looking at the human organism using the comparative, dynamic approach.

* The intermediary role between the two pairs of spatial bodies does not have to be the spherical form; this is only chosen because it provides an easy demonstration of the phenomenon. Therefore we confine ourselves here to this simpler case.

** In the case of two-dimensional figures instead of three-dimensional ones, we are dealing with the polarities of *line* and *point* rather than *plane* and *point*. (Compare the circle built up of points and the circle built up of tangent lines in Figure 5.2)

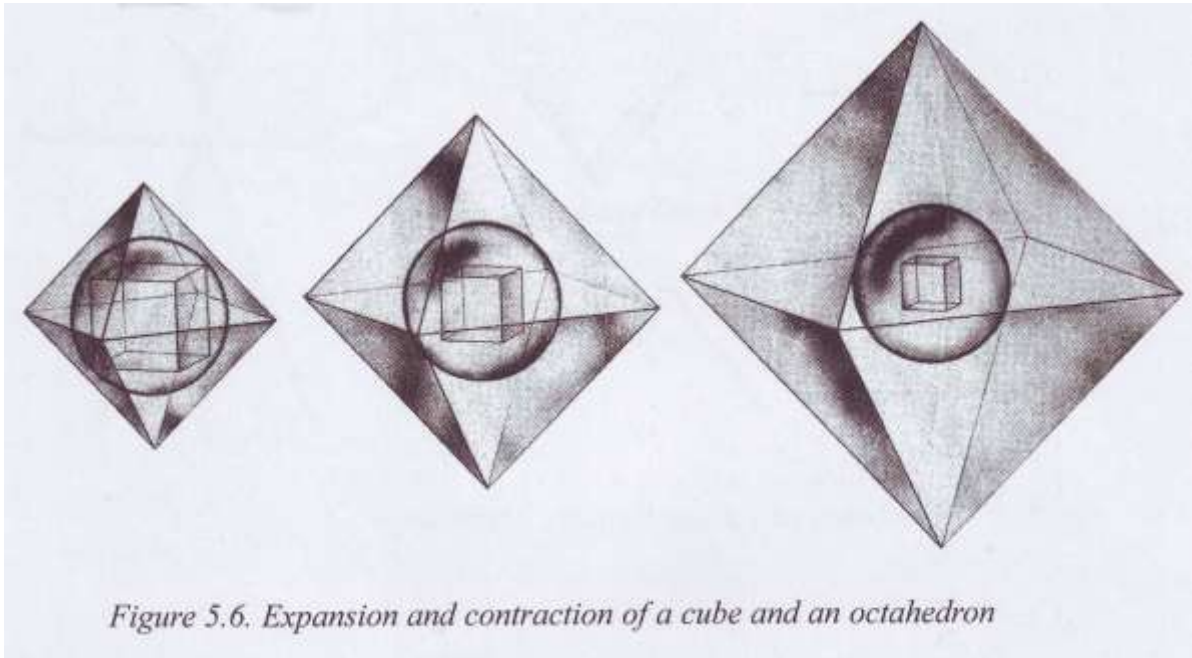


Figure 5.6. Expansion and contraction of a cube and an octahedron

The general laws of the polar relationship existing between point and plane can be demonstrated with Figure 5.7, even though the scope of this chapter does not allow us to enter into the subject deeply. Every point of the polar plane outside the sphere corresponds to a plane inside the sphere (and the other way around). This plane inside the sphere is determined by a conic section which has the point on the polar plane as its top and touches the sphere. Another point on the polar plane gives us yet another plane inside the sphere. Where the two planes intersect on the inside, they form a line. The third point on the polar plane makes yet a third plane inside the sphere, which has one point in common with the line thus formed at the intersection.*

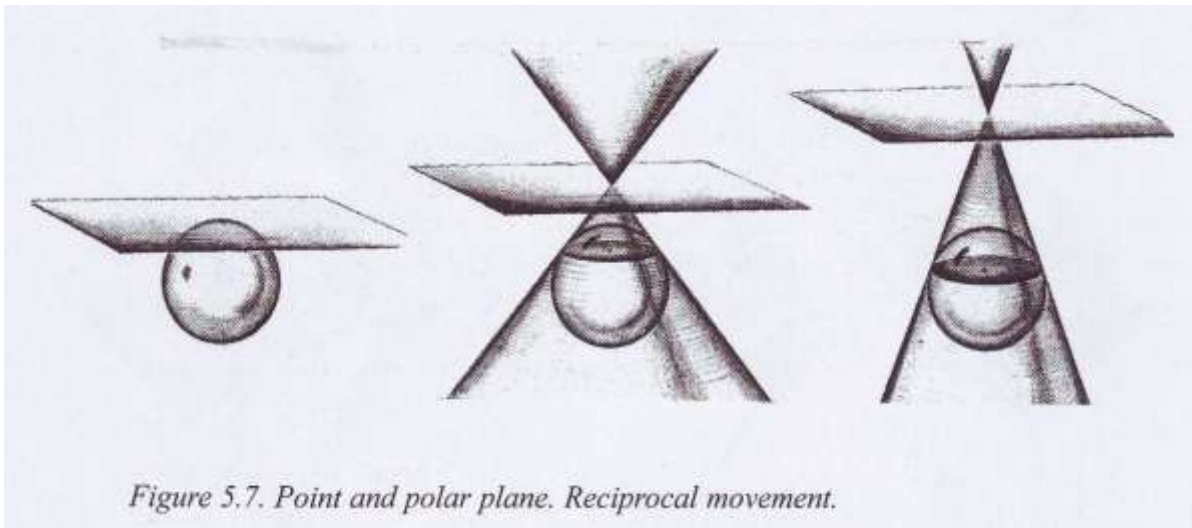


Figure 5.7. Point and polar plane. Reciprocal movement.

It is a rare occasion when this polar plane touches the sphere itself, but when it happens point and plane coincide. The polar point then lies on the polar plane. Where would the point be when the polar plane is in infinity?

Note once again, that the duality principle with regard to the exchange of point and plane applies here also. Three *points* on the polar *plane* correspond to three *planes* in the sphere. Three *points* determine the (polar) *plane* and three *planes* determine the (polar) *point* inside the sphere.

In Assignment 3 it will be obvious that the plane and point are always located on either side of the sphere. Here, too, the surface of the sphere represents a boundary which cannot be crossed, just as we saw in Section 5.1 in the case of the circle turning inside out. The sphere - even though it is itself placed in space — determines two separate worlds.

Assignment 3

Imagine a point inside the sphere and imagine its polar plane outside the sphere. What happens to the polar plane when the point moves towards the centre of the sphere, when it is at the centre of the sphere, and what happens when it moves towards the outside of the sphere, and when it touches the sphere itself.

Thus we have to look upon the intermediary sphere as the boundary between space and counterspace (positive and negative space respectively). Everything contained in one of those spaces will inevitably have its correspondent in the other in such a way that the polarity of point and plane is met. It probably goes without saying, that when there is a finite spatial figure (not an infinitely small point) within the sphere, that this can be conceived as being composed of a large number of (an infinitely large number, actually) points, each of which has a defined place and corresponds to as many planes, which form a mirror image as tangent planes on the outside. Such a shape can look quite different in certain cases, while retaining a quite definite relationship to the figure inside the sphere. Since one can think every spatial body as being built up either out of points or planes, this means that everybody in sense-perceptible (positive) space has a corresponding (but visually different) body in counterspace (negative space), and vice versa. So there is not only an inversely proportional relationship, which we mentioned before, but also a qualitative form relationship, or metamorphosis relationship. In the case of mathematically (or geometrically) defined bodies, it is still relatively easy to construct the one on the basis of the other. In principle, everybody is related to a counterpart, and that also goes for the human body! It is quite an amazing thought that every individual human being has a corresponding non-sensory body in a different space, which is not a hypothetical possibility, but a reality which can only be grasped as an idea. That body has just as many specific traits as the ones we can perceive in the physical human being with our senses

The two concepts of point and plane bear the principles within them for a new morphology. When we look around us in nature, we always encounter both in more or less recognizable form. It is a matter of recognizing the gesture, which strives towards either a plane or a point. To take an example, we shall first look at the plant. In its most complete form, when it is fully grown, it consists of leaf, Goethe said: 'it is all leaf' (*Alles ist Blatt*), (see Section 6.3). The planar element is obvious here, but other parts can also be understood as metamorphoses of leaves. The ovary, for example, comes into existence when one or more carpels grow together, and the flower also consists of leaves in all its components. When fully contracted into the seed, the plant has a shape which tends towards the point; when fully grown out, its physical form tends to the all-embracing plane. These two extreme manifestations of the plant are polar opposites of one another.* Phenomena of expansion and contraction also occur in numerous places of the human body. In many cases these will also have a polar relationship. Similarly, examples of the character of the plane, the line, and the point can be found in the skeleton. The parietal bone and the wing-shaped bone of the ileum are planar, the humerus and the femur are linear, and the bones of metacarpals and metatarsals are point-like in character.

* This applies not only to forms, but sometimes also to processes. The fully grown plant, for example, has no potential for further growth left; the seed, by contrast, is still full of potential.

The same tendencies can be recognized in the behavior and appearance of the four elements, and also in the states of aggregation of earth substances, the physical states.

The *solid* state tends to be heavy, compact, and contracted, as seen in stones. The element of earth is three-dimensional in character.

In *liquids* (water) we see the shape of the drop, but at the same time we observe planes on the surface. So it appears to have both aspects. Since water contains the opposite characteristics of both the point and the plane, it also becomes more comprehensible why it plays such a mediating role in nature. In truth there really is only one single surface (all water surfaces on earth, lakes and seas taken together form one big drop). Thus the element of water is only two-dimensional.

In *air*, the gaseous state, the characteristic tendency of evaporation and expansion is evident, even though gas is still subject to gravity. The gaseous state manifests a striving away from a point to the periphery; it has no surfaces whatsoever, only direction. Therefore its character is *one-dimensional*

Fire, the element of warmth, penetrates everything else. Here we are dealing with a 'substance' which belongs to be 'all embracing plane.' Warmth can be found in space, but it is unhampered by physical boundaries. The tendency of warmth to strive outward is irrepressible; at most it can be slowed down, as is done in a thermos flask, for example.* This tendency allows us to characterize warmth as having no dimension.

One finds such tendencies in gesture and form not only by looking for general similarities in the sense of analogies, but by living into those tendencies with such a degree of mathematical exactness, that links to corresponding natural phenomena become obvious. When we imagine form relationships such as the ones we spoke of above not as fixed states, but as stages in everlasting processes of movement, the dimension of time enters in next to the spatial dimension. We indicated above how the sequence of cause and effect which we determined in physical space is reversed in counterspace. That means nothing less than that time runs backwards there!

Imagine the following. Lift your arm. At the same time something corresponding to your arm will move in counterspace, and it will move down! **

When we look back in the evening on all that happened during the course of the day, we are faced with a medley of causes and effects. We are subjected to those, and they can often give us the sensation that we do not live but are being lived. Therefore the anthroposophical path of schooling frequently recommends the so-called *Rückschau* (looking back on the day); when carried out regularly, it will counterbalance the stream of events which come to us from out of the physical world. One of the things this accomplishes is to overcome the pressure to interpret everything in terms of cause and effect. Practice of this exercise makes it noticeably easier to approach things and events openly. It is also useful to practice with other things than daily events which run their natural course in the stream of time.

As an exercise try saying the letters of the alphabet in reverse order, for example. Even more difficult would be to sing a song backwards!

* The element of warmth is quite special. Even though it is spatial in that there are warmth bodies on earth, warmth also has a radiant aspect (infrared radiation), and in that respect it can be said to belong to counterspace also. Therefore we are justified in speaking of both physical warmth and etheric warmth.)

** Note that there is an additional factor which plays a role in such an event, namely the will. This will (or intention) originates in an even higher realm than counterspace, in the astral world. The effect, however, shows itself in the etheric world.

Projective geometry is a path of schooling which offers the researcher new insights. When one becomes truly conversant with 111k.sc newly gained insights, sensing what it is like to be a point or an expanding infinite plane, these concepts become more alive and more saturated. They will turn into something like organs of perception. This allows one to see phenomena from a different perspective than before, and to make new discoveries. It goes without saying that such a receptivity will not arise through superficial acquaintance. We can only give a fragmentary indication here. By actively drawing and constructing the figures of projective geometry one will live into the subject matter much more strongly than by looking at ready-made drawings such as those reproduced here!

5.4 Concluding remarks

In this chapter, an attempt was made to open up a different experience of space. It is possible to gain an inner sense of infinity, and of supersensory regions 'beyond' infinity. Projective geometry has an inner logic and inevitably leads to the omnipresent reality of counterspace. The two areas in which morphogenetic processes take place were introduced. The first of these areas is positive space, where the point-based forces rule;* the second, counterspace or negative space, is where the peripheral forces or cosmic forces belong. The latter is also called sculptural space by Rudolf Steiner. This is a very meaningful term, because it indicates where we actually should look for the origin of formative processes. It points us to the space of sculptural planes. We have to think first of all how shapes actually arise, for example when we shape a churl,, of clay with our hands. When we do that we mainly use the palms of our hands (planar surfaces).

If we look at nature in an imaginative, creative way, we discover that spherical shapes predominate. It could be objected that trees, for example, show a pattern of branching out, and hence a more linear tendency; but closer inspection shows us that this pattern is subject to the larger striving of the tree to make a more or less spherical shape with the crown. Very clear expressions of what we mean here may be found in more primitive growth processes, clearly visible in the growth of bacteria on a culture medium in a Petri dish; another clear expression can be found in a subdividing zygote. Both make a spherical shape growing from out of a central point. The sphere has its counterpart in negative space, which is also a sphere, but one that is inverted, or 'negative'. This primary mathematical form is a characteristic of primitive growth. A spectacular example, course, is the way a zygote (of lower vertebrates) develops through the morula stage into the blastula stage.

Now that we have come to recognize the differences and characteristics of two complementary spaces, we have a basis to recognize a highly dramatic event in the process of gastrulation: it will be immediately apparent how in one place within the spherical surface a hollow space comes into being! Does that not mean that counterspace is at work within the space of the central forces? We can observe repeated occurrences of this same gesture, for example in the formation of the neural tube and eye placode. Such a gesture of turning inside out does show that something from out of another world is being internalized, which in turn manifests perhaps in the behavior of the animal or human being.* Even higher realms are at work here, namely the astral world.

* Think of the genes, which are like 'points' in the DNA sequence. They form as many focal points for the peripheral formative forces.

In this chapter we have dealt with matters which are ostensibly far removed from everyday medical-therapeutic practice. It should be born in mind, however, that it is possible to consider changes in form and structure and physiological processes of the human body in a new way, and that is what this is about. We intentionally appealed to (mathematical) thinking, because only in thinking do we have the possibility to grasp the essence of formative processes. For thinking is akin to formative processes and growth processes.

It is of the greatest importance to know that ordinary human powers of thought *are refined powers of configuration and growth* [italics added]. A spiritual principle reveals itself in the configuration and growth of the human organism. And as life progresses this principle emerges as the spiritual power of thought.

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* The neural tube and the eye placode are both part of the nervous system, the function of which is to take in the outer world. So here we can experience a correspondence between that function on the one side and the formative gestures on the other.

** *Es ist von der allergrössten Bedeutung zu wissen, dass die gewöhnlichen Denkkräfte des Menschen die verfeinerten Gestaltungs- und Wachstumskräfte sind. Im Gestalten und Wachsen des Menschlichen Organismus offenbart sich ein Geistiges. Denn dieses Geistige erscheint darn im Lebensverlaufe als die geistige Denkkraft.* (Rudolf Steiner and Ita Wegman: *Extending Practical Medicine*, Chapter 1, page 6.)